

GENERALIZATIONS IN MATHEMATICS: FROM PRIMARY TO SECONDARY SCHOOL AND BEYOND

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Extended Abstract

The development of generalizations plays a pivotal role in fostering both mathematical thinking and mathematical growth. According to Mason (1996) generalization is the heart and soul of mathematics. If teachers are unaware of its prevalence and promise, and not in the habit of getting their learners to experiment, make conjectures, express and justify their own generalizations, then mathematical thinking becomes the worst casualty of our mathematics classroom. Furthermore, any form of justification supporting a generalization (like: external conviction, empirical, generic and deductive) invokes a particular kind of reasoning or a combination of reasoning forms (like inductive, analogical, deductive) which inevitably provides the grounds on which students can naturally question, argue and conjecture, construct a generalization, and/or also explain why a particular generalization is either true or false (compare Blanton & Kaput, 2002; De Villiers, 2003a; Ellis, 2007; Hanna, 2000).

With specific reference to learning and doing mathematics, the National Statement on Mathematics for Australian Schools contends that

“Mathematical discoveries, conjectures, generalizations, counter-examples, refutations, proofs are all part of what it means to do mathematics. School mathematics should show the intuitive and creative nature of the process, and also the false starts and blind alleys, the erroneous conceptions and errors of reasoning which tend to be a part of mathematics” (Australian Educational Council, 1991, p.14).

Indeed the cyclical processes of experimentation, conjecturing, testing, generalizing, refuting and justifying that mathematicians traverse in order to construct the polished definitions and theorems that we find in most mathematics textbooks and curriculum documents around the world are not reported on in most cases (compare De Villiers, 2004, 2010). In particular, Freudenthal (1973) talks about the way in which textbooks hide, disguise or distort the way in which real mathematics is invented, and thus argues for an approach of ‘re-invention’. Taking cognisance of all this, the new South African mathematics Curriculum through all the grades places appropriate emphasises on reasoning, conjecturing, generalizing and justifying across the content topics.

For example, the Curriculum Assessment Policy Statement (CAPS) Mathematics Senior Phase Grades 7-9, teachers are expected to give learners chances at investigations that are designed within the context of the following broad guidelines:

“Investigations promote critical and creative thinking. It can be used to discover rules or concepts and may involve inductive reasoning, identifying or testing patterns or relationships, drawing conclusions, and establishing general trends.” (p. 156).

This presentation provides plausible ideas as to how learners can construct inductive generalizations and experience heuristic counter-examples (wherever possible) that can cause (or force) them to modify (refine) their conjectures (or inductive generalizations), i.e. make a conceptual change as per Piaget’s model of socio-cognitive conflict. In addition, ways of justifying an inductive generalization are explored and discussed.

Furthermore, this presentation provides a road map of how analogical reasoning can be used to extend generalizations from one domain to the next, as well as aiding in the construction of deductive generalizations and logical explanations of particular generalizations through parallel transfer of explanatory structures from one domain across to another domain. In particular, this presentation brings to the fore three basic ways in which generalizations can be developed: namely inductive, analogical and deductive.

Establishing a learning environment in our mathematics classrooms, wherein students are given an opportunity to experience the underlying processes of experimenting, conjecturing, specializing, generalizing and justifying can enable mathematics learners to come to view mathematics as a process that is within their capabilities and not just a series of ‘products’ that are produced, and hence learn mathematics in a more meaningful way.

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